Body Density Measurement

Hans Fugal

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Abstract

This paper discusses an easy and affordable method for measuring body density, based on the principles of hydrostatic weighing but within reach of any individual with access to basic materials and a swimming pool or lake.

1 Introduction

Many people want to "lose weight", but what they really want is to "lose fat". The bathroom scale is an insufficient instrument to track fat loss. There are several means of estimating body fat, but most are infeasible for frequent use by individuals. Skinfold measurement can be unreliable, especially when self-performed by amateurs. This paper describes another method based on Hydrostatic Weighing requiring only access to a pool, a gallon jug, a scale, and a calculator.

2 Basic Method

To measure your body density (ρ), and estimate your body fat percentage (%BF), you need access to a swimming pool or other body of fresh water, a gallon jug, and a bathroom scale.

Weigh yourself on the scale before climbing into the pool. This is m.

Climb in the pool and get all the air bubbles out of your swimming suit and hair. At each following submersion step, exhale maximally as you go under water. Any extra air left in your lungs will make you appear to have more body fat than you actually do.

Exhale and submerge yourself and observe that you sink toward the bottom of the pool. If you do not sink, you will need a counterbalance (discussed later). Now take the empty gallon jug (e.g. a milk jug) and hold it under the water as you exhale and submerge. You should float to the surface even after exhaling completely. If not, you may need two jugs. We will refer to this jug or multiple jugs collectively as the "buoy" from here on.

Now, add water or air to the buoy until you no longer float to the surface nor sink to the bottom. When you just hang under the water as if you were weightless, you have achieved neutral buoyancy. Measure how much water is in the buoy, and subtract that number from the total capacity of the buoy to get the volume of air in the buoy. This number is v_{buoy}.

Now convert the measurements to kilograms and liters (1 lb = 0.4536 kg and 1 cup = 0.2366 liters) and use this formula to calculate your body density:

$$\rho = \frac{m}{\frac{m}{0.997\,\text{kg/liter}} - (v_{\text{buoy}} + 1.3\,\text{liters})}$$

Now take ρ, your measured body density, and estimate your body fat with Siri's equation[1]:

$$\%\text{BF} = \frac{495}{\rho} - 450$$

If you have done the experiment carefully and your lung capacity is about average, this answer will probably be within 2% of your true body fat.

3 Advanced Method

To get a more accurate answer, we need to account more carefully for the air that remains in your lungs even after maximal exhalation. To do this we need to measure your vital lung capacity, i.e. the amount of air you can exhale after a full inhalation.

Get two gallon jugs and fill them and turn them upside down under water. There should be no air trapped in them. Now take a full breath and exhale through a straw into one of the jugs. When it fills entirely with air, switch to the second jug (you may not need both jugs). When you have exhaled completely, measure the amount of water remaining in the jugs, and subtract from the total capacity to get the amount of air that you exhaled. This amount is your vital lung capacity (VC). You should then adjust this measurement to body temperature, pressure, water vapor saturated (BTPS). It seems that a constant correction of 2 to 4% is appropriate (multiply by 1.03) [2]. Now estimate your residual lung capacity (RV) as $\text{RV} = \frac{x}{100-x}\text{VC}$ where $x = 14.0 + \text{age} \times 0.39$ if you are male, or $x = 19.0 + \text{age} \times 0.34$ if you are female[3]. The estimate of the ratio $x = {}^{\text{RV}}/_{\text{TLC}}\%$ is $\pm 11\%$, so this is more like a wild guess than an estimate, but it's about the best you can do without measuring RV.

If you do not sink when exhaling completely, you will need a counterbalance. Find a heavy object (5 lbs would be a good size) which you can bring under the water with you and the buoy. Measure its volume by submerging it in water and measuring the displacement (as Archimedes did with the King's crown). This is v_c. Measure its mass by weighing it on a scale. This is m_c.

Now use the following equation to calculate your body density:

$$\rho = \frac{m_{\text{body}}}{\frac{m_{\text{body}} + m_c}{0.997\,\text{kg/liter}} - (v_{\text{buoy}} + \text{RV} + 0.1\,\text{liters} + v_c)}$$

Use Siri's equation again to calculate %BF. The 0.1 liters term is an estimate of gastrointestinal gasses. For a more accurate result do this in the morning before breakfast, to minimize the amount of non-body mass in your system.

4 The Physics

$$
\begin{aligned}
F_{\text{buoyant}} &= \rho_w v g && \text{Archimedes' Principle} \\
w &= mg \\
w &= F_{\text{buoyant}} && \text{at neutral buoyancy} \\
mg &= \rho_w v g \\
m &= \rho_w v \\
v &= \frac{m}{\rho_w} \\
v_b + v_a + v_c &= \frac{m_b + m_c}{\rho_w} && \text{ignoring the mass of air} \\
v_b &= \frac{m_b + m_c}{\rho_w} - v_a - v_c \\
\rho_b &= \frac{m_b}{v_b} \\
\rho_b &= \frac{m_b}{\frac{m_b + m_c}{\rho_w} - v_a - v_c} \\
v_a &= v_{\text{buoy}} + \text{RV} + v_{\text{GI}} \\
\overline{\text{RV}} &= 1.2\,\text{liters}[4] \\
v_{\text{GI}} &\approx 0.1\,\text{liters}[5] \\
\rho_w &= 0.997 \pm 0.001\,{}^{\text{kg}}/_{\text{liter}} && \text{at standard pool temperatures}
\end{aligned}
$$

5 Uncertainty Analysis

Considering the following uncertainties for the measurements:

$$
\begin{gathered}
m_b \pm 0.02\,\text{kg} \\
m_c \pm 0.02\,\text{kg} \\
\rho_w \pm 0.001\,{}^{\text{kg}}/_{\text{liter}} \\
v_a \pm 0.1\,\text{liter} \\
v_c \pm 0.01\,\text{liter} \\
v_{\text{GI}} \pm 0.075\,\text{liter} \\
\text{RV} \pm 0.5\,\text{liter}
\end{gathered}
$$

The uncertainty in estimating body fat is between 2% and 5% (depending on the weight of the subject)[6]. The biggest factor is the uncertainty in estimating RV. If RV were known with high precision, uncertainty would drop to 0.75% to 1%.

6 Conclusion

The ease and accuracy of this method is comparable to using calipers to measure skinfold density. The cost for equipment is lower, but this method is less convenient than calipers except for swimmers who already frequent the pool. If one can manage to get an accurate measure of RV (e.g. while participating in hydrostatic weighing) the accuracy is higher than that of skinfold measurement.

Either method is preferable to using the BMI equation, which does not take body composition into account.

References

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